Lower Bounds for Evaluating Schedule Performance in Flexible Job Shops

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Abstract

In this paper, we are interested in the multiobjective evaluation of the schedule performance in the flexible job shops. The Flexible Job Shop Scheduling Problem (FJSP) is known in the litteratue as one of the hardest combinatorial optimization problems and presents many objectives to be optimized. In this way, we aim to determine a set of lower bounds for certain criteria which will be able to characterize the feasible solutions of such a problem. The studied criteria are the following: the makespan, the workload of the critical machine, and the total workload of all the machines. Our study relates to the determination of a practical method in order to evaluate the representative performance of the production system.

Keywords

Performance evaluation, lower bounds, flexible job shop scheduling problems, multiobjective optimization.

I. Introduction

The flexible job shop scheduling problem is a problem of planning and organization of a set of tasks to be performed on a set of resources with variable performances [1], [2], [3], [4], [5]. In the literature, the authors generally consider two steps in its resolution [4]. The first one is to assign the various tasks to the suitable resources. The second step is the sequencing of the tasks and the computation of the starting times by taking account of the various constraints of precedence and resources. Nevertheless, Dauzère-Pérès et al have recently proposed an interesting method to solve this problem by considering the two steps at the same time [5]. Morover, in [6], Dauzère-Pérès et al have given an extension of this problem in which tasks can be performed by several resources at the same time. Many criteria can be considered in the resolution of such a problem. Mainly, two objectives could be distinguished. The first one is to balance the workloads of the machines, the second one is to organize the various tasks in minimizing the overall completion times. This variety of criteria induces additionnal difficulties related to the evaluation of the feasible solutions as well as the comparison between the resolution methods.

In this way, we aim in this paper to contribute in the reduction of such difficulties by proposing a set of lower bounds for some representative criteria of such a problem. In addition, we propose to undervalue the criteria related to the workload of the resources. The second part of this article presents the specificities of the flexible job shops and gives the mathematical formulation used to deal with such a problem. Thus in section III, we describe the various steps followed in the calculation of the lower bounds proposed. The last part will be devoted to the presentation of some results and some conclusions concerning this research work.

II. Problem Formulation

The problem is to organize the execution of N jobs on M machines. The set of machines is noted U. Each job J_j represents a number of n_j non preemptable ordered operations (precedence constraint). The execution of the i^{th} operation of job J_j (noted $O_{i,j}$) requires one resource or machine selected from a set of available machines. The assignment of the operation $O_{i,j}$ to the machine M_k entails the occupation of this machine during a processing time called $d_{i,j,k}$. Thus, to each FJSP (Flexible Job Shop Scheduling Problem), we can associate a table D of processing times such that: $D = \{d_{i,j,k} \in IN^* \mid 1 \leq j \leq N; 1 \leq i \leq n_j; 1 \leq k \leq M\}$.

In this problem, we make the following hypotheses:

- all machines are available at $t = 0$ and each job J_j can be started at $t = r_j$,

- at a given time, a machine can only execute one operation: it becomes available to other operations once the operation which is currently assigned to is completed (resource constraints),

- to each operation $O_{i,j}$, we associate an earliest starting time $r_{i,j}$ calculated by the following formula:

$r_{1,j} = r_j \ \forall\ 1 \leq j \leq N$, and $r_{i+1,j} = r_{i,j} + \gamma_{i,j} \ \ \forall\ 1 \leq i \leq n_j - 1,\ \forall\ 1 \leq j \leq N$.

where $\gamma_{i,j} = \min_k (d_{i,j,k}) \quad \forall\ 1 \leq i \leq n_j - 1, \forall\ 1 \leq j \leq N$.

The FJSPs present two difficulties. The first one is to assign each operation $O_{i,j}$ to a machine M_k (selected from the set U). The second one is the computation of the starting time $t_{i,j}$ and the completion time $tf_{i,j}$ of each operation $O_{i,j}$.

The considered objective is to minimize the following criteria:

- the makespan:

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$$Cr_1 = \max_j \{tf_{n_j,j}\} \tag{1}$$

- the workload of the most loaded machine:

$$Cr_2 = \max_k \{W_k\} \tag{2}$$

where W_k is the workload of M_k,
- the total workload of the machines:

$$Cr_3 = \sum_k W_k \tag{3}$$

Definition 1: To each repartition of the operations on the resources set, we associate an assignment. Each assignment is characterized by a set S such that: $S = \{S_{i,j,k} \in \{0,1\} \, | 1 \leq j \leq N, 1 \leq i \leq n_j, 1 \leq k \leq M\}$. $S_{i,j,k} = 1$ if $O_{i,j}$ is assigned to M_k else $S_{i,j,k} = 0$.

Definition 2: $\widetilde{E}$ is a numerical function defined as follows: $\widetilde{E}(x) = x$ if x is integer else $\widetilde{E}(x) = E(x) + 1$, where $E(x)$ is the integer part of x.

III. New Lower Bounds

The problem of the lower bounds has been considered in the literature for many scheduling problems, in particular, the one-machine problem [7], the parallel machines problrm [8], [9], the hybrid flow-shop problem [11] and the job-shop problem [12]. Generally, the methods suggested are based on the constraint relaxation (preemption of the tasks, disjunctive constraint on the resources...) in order to estimate the makespan of the optimal schedule. In this paper, we generalize some results proposed in the literature for the parallel machines problem [9] and we propose others new considerations based on the evaluation of the cost of assigning a certain number of tasks to certain resources.

A. Lower Bound for the Workload of the Machines

The total workload is equal to the sum of all the processing times of all the operations carried out by the set of the machines according to the chosen assignment. In addition, for each operation $O_{i,j}$, the processing time is superior or equal to $\gamma_{i,j}$, therefore:

$$Cr_3 \geq \sum_j \sum_i \gamma_{i,j}$$

B. Lower Bound for the Workload of the Critical Machine

Lemma 1: $\widetilde{E}\left(\frac{\sum_j \sum_i \gamma_{i,j}}{M}\right)$ is a lower bound of Cr_2.

Proof: Trivial, the workload of the critical machine is higher than the mean of the workloads. ■

Now, let consider the operations carried out by the set of the machines. $\widehat{N}$ is the average of the operation numbers carried out by one machine ($\widehat{N} = \frac{1}{M}.\sum_j n_j$). Then, we have at least one machine M_{k_0} that will perform at least N_{k_0} operations such that: $N_{k_0} \geq \widetilde{N} = \widetilde{E}\left(\widehat{N}\right)$. Thus, to each machine M_k, we associate the $\widetilde{N}$ shortest operations that it can perform. $D_{k,\widetilde{N}}$ is the sum of the processing times of these associated operations and $\delta_{k_0,\widetilde{N}}$ represents the minimal value of these sums when varying k:

$$\delta_{k_0,\widetilde{N}} = \min_{1<k<M} \left(D_{k,\widetilde{N}}\right)$$

The critical machine will have a workload more important than $\delta_{k_0,\widetilde{N}}$, then, we obtain the following lemma.

Lemma 2: $\delta_{k_0,\widetilde{N}}$ is a lower bound of Cr_2.

Definition 3: $a(k, N\prime)$ is a binary variable which is true if the machine M_k carries out $N\prime$ operations among the NT operations (NT is the total number of the operations and $N\prime \leq NT$).

Let us suppose that $a(k, N\prime) = 1$, then the machine M_k will work during a time at least equal to $D_{k,N\prime}$ ($D_{k,N\prime}$ is the sum of the $N\prime$ shortest processing times of the operations that we can perform on M_k). In this case, the remainder of the operations (the $NT - N\prime$ operations) must be carried out on $U^k = U - \{M_k\}$. This obligation can be regarded as a scheduling problem of $(NT - N\prime)$ operations on the $(M-1)$ machines of U^k. Therefore, for such a problem, we can apply the result of lemma 1 to undervalue the workload of the most loaded machine of U^k. Thus, this workload is superior or equal to $\alpha_{k,N\prime} = \widetilde{E}\left(\frac{\Gamma_{k,N\prime}}{M-1}\right)$, where $\Gamma_{k,N\prime}$ is the sum of the $(NT - N\prime)$ smallest values of $\gamma_{i,j}^k$ such that $\gamma_{i,j}^k = \min_{\substack{1<h<M \\ h\neq k}} (d_{i,j,h})$. Then, we obtain the following lemma.

Lemma 3: If $a(k, N\prime) = 1$, then there exists a machine for which the workload is at least equal to $\beta_{k,N\prime}$ such that:

$$\beta_{k,N\prime} = \max\left(D_{k,N\prime}, \alpha_{k,N\prime}\right)$$

Lemma 4: $\mu_{k_0,\widetilde{N}} = \min_{1<k<M}\left(\min_{N\prime \geq \widetilde{N}}\left(\beta_{k,N\prime}\right)\right)$ is a lower bound of Cr_2 and it is superior to $\delta_{k_0,\widetilde{N}}$.

Proof: Obvious, according to lemma 3 and the definition of $\widetilde{N}$. ■

Using lemmas 1 and 4, we deduce the following lemma.

Lemma 5:

$$Cr_2 \geq \max\left(\widetilde{E}\left(\frac{\sum_j \sum_i \gamma_{i,j}}{M}\right), \mu_{k_0,\widetilde{N}}\right)$$

C. Lower Bound for the Makespan

Lemma 6:

$$Cr_1 \geq \max_j\left(r_j + \sum_i \gamma_{i,j}\right)$$

Proof: Obvious, because of precedence constraints on the operations over the jobs. ■

Lemma 7: Let R_M the sum of the M smallest release dates $r_{i,j}$ of the operations, then:

$$Cr_1 \geq \widetilde{E}\left(\frac{R_M + \sum_j \sum_i \gamma_{i,j}}{M}\right)$$

Proof: This demonstration is inspired of [9]. In 1987, Carlier have proposed an interesting lower bound for the problem of n operations on m identical machines in minimizing the makespan. This bound takes account of the release date of each operation and shows that the activity intervals of the machines, in the best case, have the following form: $[r_{i_k}, f^*]\ \forall\ 1 \leq k \leq m$ where f^* is the optimal value of the makespan and $[r_{i_1}, r_{i_2}, ..., r_{i_m}]$ the m smallest release dates of the operations. By using the same idea in our problem, we obtain the following relation: $Cr_1 \geq \frac{R_M + Cr_3}{M}$. Moreover, we have $Cr_3 \geq \sum_j \sum_i \gamma_{i,j}$ and the makespan is integer, therefore, the lemma is justified. ■

Definition 4: Let $E_{N\prime}$ the set of the combinations constituded of $N\prime$ operations among the NT operations and $C_{N\prime}$ an element of $E_{N\prime}$: $C_{N\prime} = \{O_{i_1,j_1}, O_{i_2,j_2}, ..., O_{i_{N\prime},j_{N\prime}}\}$. We define $T_{k,C_{N\prime}}$ as follows:

$$T_{k,C_{N\prime}} = \min_{1<q<N\prime}\left\{r_{i_q,j_q}\right\} + \sum_{1<q<N\prime} d_{i_q,j_q,k}$$

Proposition 1: $T_{k,N\prime}$ is a lower bound of the value of the date at which the machine M_k can carry out $N\prime$ operations.

$$T_{k,N\prime} = \min_{C_{N\prime} \in E_{N\prime}}\left\{T_{k,C_{N\prime}}\right\}$$

Proof: For a given combination $C_{N\prime}$ of $E_{N\prime}$, the machine M_k can start to perform operations at least at the date $t = \min_{1<q<N\prime}\left\{r_{i_q,j_q}\right\}$ and must work for a duration at least equal to $\sum_{1<q<N\prime} d_{i_q,j_q,k}$. ■

Lemma 8: Let V_z the subset of the operations set defined as follows: $V_z = \{O_{i_{z+1},j_{z+1}}, O_{i_{z+2},j_{z+2}}, ..., O_{i_{NT},j_{NT}}\}$ such that $r_{i_{z+1},j_{z+1}} \leq r_{i_{z+2},j_{z+2}} \leq ... \leq r_{i_{NT},j_{NT}}$, then:

$$T_{k,N\prime} = \min_{1<z<NT-N\prime+1} \left(r_{i_z,j_z} + d_{i_z,j_z,k} + \min_{C'_{z,N\prime} \in E'_{z,N\prime}} \left(\Delta^k \left(C'_{z,N\prime} \right) \right) \right)$$

where: $\Delta^k \left(C'_{z,N\prime} \right)$ is the sum of the processing times of the operations of $C'_{z,N\prime}$ on M_k; $C'_{z,N\prime}$ is an element of $E'_{z,N\prime}$ and $E'_{z,N\prime}$ is the set of the combinations of $(N\prime - 1)$ operations chosen among the $(NT - z)$ operations of V_z for $z \in \{1, 2, ..., N_t - N\prime + 1\}$.

Proof: For proof, see [10]. ■

Let us suppose that $a(k, N\prime) = 1$, then the machine M_k will work until a date at least equal to $T_{k,N\prime}$. In this case, the remainder of the operations (the $NT - N\prime$ operations) must be carried out on U^k. This obligation can be regarded as a scheduling problem of $(NT - N\prime)$ operations on the $(M - 1)$ machines of U^k. Therefore, for such a problem, we can apply the result of lemma 7 to undervalue the necessary time for such an execution (the minorant of such necessary time is noted $\lambda_{k,N\prime}$). Thus, if $NT - N\prime \geq M - 1$, $\lambda_{k,N\prime} = \widetilde{E}\left(\frac{R_{M-1} + \Gamma_{k,N\prime}}{M-1}\right)$ where R_{M-1} is the sum of the $(M - 1)$ smallest values of $r_{i,j}$. In the contrary case and if $NT - N\prime < M - 1$, it is more interesting to undervalue the time in question by $\widetilde{E}\left(\frac{R_{NT-N\prime} + \Gamma_{k,N\prime}}{NT-N\prime}\right)$ where $R_{NT-N\prime}$ is the sum of the $NT - N\prime$ smallest values of $r_{i,j}$. Therfore, we have the following lemma.

Lemma 9: If $a(k, N\prime) = 1$, then we need to perform operations until a date at least equal to $\tau_{k,N\prime}$ such that:

$$\tau_{k,N\prime} = \max\left(T_{k,N\prime}, \lambda_{k,N\prime}\right)$$

with:

$$\lambda_{k,N\prime} = \widetilde{E}\left(\frac{R_{M-1} + \Gamma_{k,N\prime}}{M-1}\right) \text{ if } NT - N\prime \geq M - 1,$$

$$\lambda_{k,N\prime} = \widetilde{E}\left(\frac{R_{NT-N\prime} + \Gamma_{k,N\prime}}{NT - N\prime}\right) \text{ if } NT - N\prime < M - 1.$$

Theorem 1: $\tau_{k_0,\widetilde{N}}$ is a lower bound of Cr_1:

$$\tau_{k_0,\widetilde{N}} = \min_{1<k<M} \left\{ \min_{N\prime \geq \widetilde{N}} \{\tau_{k,N\prime}\} \right\}$$

Proof: $\min_{N\prime \geq \widetilde{N}} \{\tau_{k,N\prime}\}$ represent a lower bound of the makespan if the machine M_k carries out a number of operations superior or equal to $\widetilde{N}$. In addition, there is at least one machine that verify such a condition, therefore, the theorem is justified. ■

Let V the set of all the operations classified in the ascending order according to the values of $r_{i,j}$:

$$V = \{O_{i_1,j_1}, O_{i_2,j_2}, ..., O_{i_{NT},j_{NT}}\} \text{ such that } r_{i_1,j_1} \leq r_{i_2,j_2} \leq ... \leq r_{i_{NT},j_{NT}}$$

It is clear that any lower bound of the scheduling problem of V_z is also a lower bound for the initial problem (the scheduling of V).

Lemma 10: $LB2$ is an improvement of the lower bound proposed in lemma 7:

$$LB2 = \max_{1<q<NT-M} \left\{ \frac{1}{M} \cdot \left(\sum_{h=q}^{h=q+M-1} r_{i_h,j_h} + \sum_{h=q}^{h=NT} \gamma_{i_h,j_h} \right) \right\}$$

Proof: Such a result is justified by applying lemma 7 to the subsets V_z for $1 \leq z \leq NT - M - 1$. ■

Corollary 1 : Using the preceding theorems and the preceding lemmas, we obtain the folowing relation:

$$Cr_1 \geq \max\left(\max_j\left(r_j + \sum_i \gamma_{i,j}\right), LB2, \tau_{k_0,\widetilde{N}}\right)$$

*Remark*1 : In a previous work [13], we have showed that it exists an equivalence between a flexible job-shop with release dates and a flexible job-shop without release dates. Thus, we can use this equivalence to find other relations as the following relation:

$$Cr_1 \geq \max\left(\widetilde{E}\left(\frac{\sum_j r_j + \sum_j \sum_i \gamma_{i,j}}{N+M}\right), \delta_{k_0,\widetilde{N}\prime}\right)$$

with $\widetilde{N}\prime = \widetilde{E}\left(\frac{N+NT}{N+M}\right)$, $\delta_{k_0,\widetilde{N}\prime} = \min_{1<k<M}\left(D_{k,\widetilde{N}\prime}\right)$ and $D_{k,\widetilde{N}\prime}$ the function previously defined in the preceding subsection, but one can easily show that such bounds are less interesting than those defined by the preceding corollary.

D. Recapitulation

The preceding minorations will enable us to compute some limits for the values corresponding to the three criteria considered. These limits are defined by the following relations:

$$Cr_1 \geq Cr_1^*, Cr_2 \geq Cr_2^* \text{ and } Cr_3 \geq Cr_3^*$$

with:

$$Cr_1^* = \max\left(\max_j\left(r_j + \sum_i \gamma_{i,j}\right), LB2, \tau_{k_0,\widetilde{N}}\right),$$

$$Cr_2^* = \max\left(\widetilde{E}\left(\frac{\sum_j \sum_i \gamma_{i,j}}{M}\right), \mu_{k_0,\widetilde{N}}\right)$$

$$\text{and } Cr_3^* = \sum_j \sum_i \gamma_{i,j}.$$

E. Complexity

The formulas of the different lower bounds are implemented according to the corresponding algorithms. All these algorithms are polynomial. Theirs algorithmic complexities are presented in Table I.

IV. Simulation Results

To test the efficiency of the lower bounds in the evaluation of the system performance, many computational experiments have been carried out. This test consists in applying a Controlled Evolutionary Approach (CEA) [2]. The objective of such simulations is not to evaluate the efficiency of the CEA (in fact, the performance of such a method has been demonstrated in previous publications [2][14]). The objective considered is to measure the quality of the lower bounds proposed by comparing them to the various values of the criteria associated to the solutions given by the evoked method. In this section, we give a short description of the CEA, then, we present the considered examples and we finish by comparing the criteria values of the obtained solutions to the lower bounds values.

TABLE I

Lower bounds complexities

Lower bound	Complexity
Cr_1^*	$O\left(NT^2\right)$
Cr_2^*	$O\left(NT^2\right)$
Cr_3^*	$O\left(NT\right)$

A. Used Method

The problem considered presents two main difficulties. The first one is the assignment of each operation to the suitable machine. The second difficulty is the calculation of the starting times $t_{i,j}$ of each operation $O_{i,j}$. To solve such a problem, we apply initially an Approach by Localization [2]. This approach is a heuristic which makes it possible to assign the operations to the machines by taking account of the processing times and the workloads of the machines to which we have already assigned operations. Then, it makes it possible to solve the problem of the tasks sequencing thanks to an algorithm called "Scheduling Algorithm" [2] which calculates the starting times $t_{i,j}$ by taking into account the availabilities of the machines and the precedence constraints. The conflicts are solved by using traditional priority rules (SPT, LPT, FIFO, LIFO, FIRO [15], [16]), thus, we obtain a set of schedules according to the used priority rules. To such a set, we apply an evolutionary approach which is based on the schemata theorem introduced in the genetic algorithms field. Such an approach consists in the design of an assignment model which will be useful to construct the set of the new individuals. The objective is to integrate the good qualities contained in the schemata [2], [17], [18] in order to make the evolutionary algorithm more effecient and more rapid. In fact, the construction of the solutions is done by giving the priority to the reproduction of the individuals respecting the model generated by the assignment schemata and not starting from the whole set of the chromosomes (for further details, the reader is invited to consult [2] and [18]). The multiobjective evaluation of the solutions is carried out using a fuzzy Pareto approach [14]. Such an approach is based on the weighted aggregation of the different objective functions at each iteration of the evolutionary algorithm. The particularity of such an approach consists in the fuzzy computation of weights by giving the priority to the objective functions which the values are far from the corresponding lower bound value [14].

B. Results

Many series of examples have been tested to evaluate the quality of the lower bounds based on practical data. As an example, the reader could consult the simulation results of some instances at the web address: http://www.ec-lille.fr/~kacem/testsPareto.pdf. These instances come from the literature [4], [14] and present problems with 4 to 25 jobs, generally using 10 machines with 12 to 75 operations with total flexibility. The various results are summarized in Table II. For each instance, we present the values of the different lower bounds and the values of the criteria for the Pareto optimal solutions obtained by our fuzzy evolutionary approach.

The results presented in Table II show that the solutions obtained are generally very close to the optimal one. In the case of the Parallel Machines problem (a particular case of the flexible job-shop described by the instances I_5, I_6, I_7, I_8, I_9 and I_{10}), we obtain no distance between the lower bounds values and the different obtained solutions. In the general case, the small distance that we can have is due to the difficulty of multiobjective optimization which considers several nonhomogeneous and antagonistic criteria at the same time [19]. Recently, we have also tested some instances coming from the benchmarks of Hurink [20]. The results obtained confirm the good quality of the proposed lower bounds.

V. Conclusion

In this paper, we have proposed a set of lower bounds to make it easier the multiobjective evaluation problem of a schedule performance in the case of flexible job shops. These lower bounds make it possible to estimate precise limits for the optimal values of the corresponding criteria. In fact, different simulations show that the little distance between such limits and the values of the criteria obtained for the solutions generated by the evolutionary fuzzy approach is generally satisfactory and promising. This result is very interesting to facilitate the study of others multiobjective concepts (Uniform Design concept [19]) that we will consider as perspective in our future work.

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TABLE II

SOME SIMULATION RESULTS

Instances $I_{h\ h<10}$	Lower-bounds Cr_1^*	Cr_2^*	Cr_3^*	Obtained results Cr_1	Cr_2	Cr_3
I_1	16	7	32	18	8	32
				18	7	33
				16	9	35
				16	10	34
I_2	15	9	60	15	11	61
				16	10	66
				16	12	60
				17	10	64
				18	10	63
I_3	23	10	91	24	11	91
				23	11	95
I_4	7	5	41	7	5	45
				8	7	41
				8	5	42
I_5	17	13	63	17	13	63
I_6	26	26	126	26	26	126
I_7	51	51	252	51	51	252
I_8	16	13	63	16	13	63
I_9	38	38	189	38	38	189
I_{10}	63	63	315	63	63	315

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